

Some symmetry properties of spin currents and spin polarizations in multi-terminal mesoscopic spin-orbit coupled systems

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We study theoretically some symmetry properties of spin currents and spin polarizations in multi-terminal mesoscopic spin-orbit coupled systems. Based on a scattering wave function approach, we show rigorously that in the equilibrium state no finite spin polarizations can exist in a multi-terminal mesoscopic spin-orbit coupled system (both in the leads and in the spin-orbit coupled region) and also no finite equilibrium terminal spin currents can exist. By use of a typical two-terminal mesoscopic spin-orbit coupled system as the example, we show explicitly that the nonequilibrium terminal spin currents in a multi-terminal mesoscopic spin-orbit coupled system are non-conservative in general. This non-conservation of terminal spin currents is not caused by the use of an improper definition of spin current but is intrinsic to spin-dependent transports in mesoscopic spin-orbit coupled systems. We also show that the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of *finite* length of a two-dimensional electronic system with intrinsic spin-orbit coupling may be non-antisymmetric in general, which implies that some cautions may need to be taken when attributing the occurrence of nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in such a system to an intrinsic spin Hall effect.

PACS numbers: 72.25.-b, 73.23.-b, 75.47.-m

I. INTRODUCTION

The efficient generation of finite spin polarizations and/or spin-polarized currents in paramagnetic semiconductors by all-electrical means is one of the principal challenges in semiconductor spin-based electronics.^{1,2} For this purpose several interesting ideas have been proposed based on the spin-orbit (SO) interaction character of electrons in some semiconductor systems.^{3,4,5,6} One such interesting idea is the so-called *intrinsic spin Hall effect*,^{5,6} which is the generation of a finite spin current perpendicular to an applied charge current in a paramagnetic semiconductor with intrinsic SO coupling. Such ideas have attracted much theoretical interests recently⁷⁻²⁴ and substantial achievements do have been obtained along these lines,^{25,26} while at the same time they also raised a lot of debates and controversies.^{8,9,10} A central problem related to these debates and controversies is that, what is the correct definition of spin current in a system with strong SO coupling and what is the actual relation between the spin current and the induced spin-accumulation in such a system.²⁷ In most recent studies the conventional (*standard*) definition of spin current (i.e., the expectation value of the product of spin and velocity operators) have been applied. As is well know, this conventional definition of spin current can describe properly spin-polarized transport in a system without intrinsic SO coupling. However, since spin is not a conserved quantity in a system with intrinsic SO coupling, the physical meanings of spin current calculated based on the conventional definition are some-

what ambiguous and the actual relations between the spin current and the induced spin-accumulation are not much clear. In fact, as has been noticed in several recent papers,^{28,29,30,31} there may exist some serious problems with this conventional definition of spin current when using it to describe spin-polarized transport in a system with intrinsic SO coupling. In order to avoid such serious problems, several alternative definitions of spin current were proposed in these papers based on different theoretical considerations, which are significantly different from the conventional one and also significantly different from each other.^{28,29,30,31} Another possible way to circumvent this problem is to study a *mesoscopic* SO coupled system attached to external leads. If no SO couplings present in the leads or the SO couplings in the leads are much weak, then the conventional definition of spin current can be well applied without ambiguities in the leads. Several recent works have adopted this strategy^{14,15,16,17,18} and some interesting results were also obtained. Of course, the study of spin transport in mesoscopic SO coupled systems is not only of theoretical interest but also might find some practical applications in the design of spin-based electronic devices.³

In this paper we study theoretically some interesting problems related to spin-dependent transports in multi-terminal mesoscopic SO coupled systems. We focus our study on the symmetry properties of equilibrium and nonequilibrium spin currents and spin polarizations in such mesoscopic structures. As is well known, symmetry analysis is usually of great theoretical importance in the study of many physical phenomena, including

the spin-dependent transport phenomena in SO coupled systems.³² Based on the analyses of symmetry properties of equilibrium and nonequilibrium spin currents and spin polarizations in multi-terminal mesoscopic SO coupled systems, some controversial issues related to spin-dependent transports in mesoscopic SO coupled systems will be investigated in some detail in this paper. Some symmetry properties discussed in this paper might also be helpful for clarifying some controversial issues encountered in the study of spin-dependent transports in macroscopic SO coupled systems. The study carried out in this paper is based on a scattering wave function approach within the framework of the standard Landauer-Büttiker's formalism. From the theoretical points of view, this scattering wave function approach is in principle exactly equivalent to the more frequently employed Green's function approach in literature³³. The main merit of this scattering wave function approach is its conceptual simplicity, and due to its conceptual simplicity, some symmetry properties of equilibrium and nonequilibrium spin currents and spin polarizations in a multi-terminal mesoscopic SO coupled system can be more explicitly shown.

The paper is organized as follows: In section II we will first give a brief introduction of the structure considered and the approach applied. In section III we will use the approach introduced in section II to investigate whether there can exist nonvanishing equilibrium spin polarizations or nonvanishing equilibrium terminal spin currents in a multi-terminal mesoscopic SO coupled system. In section IV we will study the symmetry properties of nonequilibrium spin polarizations and nonequilibrium terminal spin currents in a typical two-terminal mesoscopic structure with both Rashba and Dresselhaus SO coupling. Finally in Section V a brief summary of the main conclusions obtained in the paper will be given.

II. DESCRIPTION OF THE STRUCTURE AND THE SCATTERING WAVE FUNCTION APPROACH

We consider a general multi-terminal mesoscopic structure as shown in Fig.1, where a SO coupled mesoscopic system is attached to several ideal leads. In a discrete representation, both the SO coupled region and the ideal leads are described by a tight-binding (TB) Hamiltonian, and the total Hamiltonian for the entire structure reads:

$$\hat{H} = H_{leads} + H_{sys} + H_{s-l}. \quad (1)$$

Here $H_{leads} = \sum_p H_p$ and $H_p = -t_p \sum_{\langle \mathbf{p}_i, \mathbf{p}_j \rangle \sigma} (\hat{C}_{\mathbf{p}_i, \sigma}^\dagger \hat{C}_{\mathbf{p}_j, \sigma} + H.C.)$ is the Hamiltonian for an isolated lead p , with $\hat{C}_{\mathbf{p}_j, \sigma}$ denoting the annihilation operator of electrons with spin index σ at a lattice site \mathbf{p}_j in lead p and t_p the hopping parameter between two nearest-neighbored lattice sites \mathbf{p}_i and \mathbf{p}_j

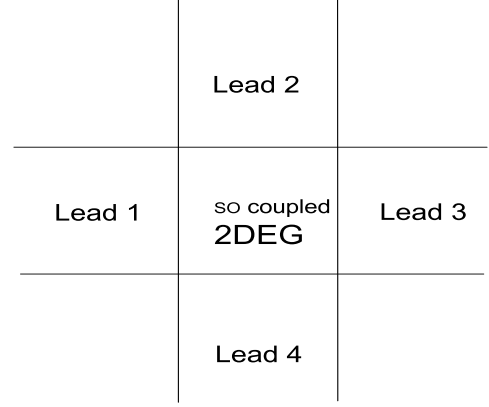


FIG. 1: Schematic geometry of a multi-terminal mesoscopic SO coupled system.

in the lead. We assume that the external leads are ideal and nonmagnetic, i.e., no any SO couplings (or other kinds of spin-flip processes rather than that induced by the scatterings from the central SO coupled region) present in the leads. In such ideal cases the standard definition of spin current can be well applied without ambiguities in the leads. Usually (if not specified) we will choose the z axis (normal to the 2DEG plane) as the quantization axis of spin. $H_{sys} = H_0 + H_{so}$ is the Hamiltonian for the isolated SO coupled region, in which H_{so} describes the SO coupling of electrons and $H_0 = -t \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle \sigma} (\hat{C}_{\mathbf{r}_i, \sigma}^\dagger \hat{C}_{\mathbf{r}_j, \sigma} + H.C.)$ describes the spin-independent hopping of electrons between nearest-neighbored lattice sites (denoted by $\langle \mathbf{r}_i, \mathbf{r}_j \rangle$) in the region. The discrete version of H_{so} will depend on the actual form of the SO interaction, e.g., for the usual Rashba and k-linear Dresselhaus SO coupling, one has

$$H_{so}^R = -t_R \sum_{\mathbf{r}_i} [i(\hat{\Psi}_{\mathbf{r}_i}^\dagger \sigma^x \hat{\Psi}_{\mathbf{r}_i + \Delta_y} - \hat{\Psi}_{\mathbf{r}_i}^\dagger \sigma^y \hat{\Psi}_{\mathbf{r}_i + \Delta_x}) + H.C.], \quad (2a)$$

$$H_{so}^D = -t_D \sum_{\mathbf{r}_i} [i(\hat{\Psi}_{\mathbf{r}_i}^\dagger \sigma^y \hat{\Psi}_{\mathbf{r}_i + \Delta_y} - \hat{\Psi}_{\mathbf{r}_i}^\dagger \sigma^x \hat{\Psi}_{\mathbf{r}_i + \Delta_x}) + H.C.], \quad (2b)$$

where t_R and t_D are the Rashba and Dresselhaus SO coupling strength, respectively, $\hat{\Psi}_{\mathbf{r}_i} = (\hat{C}_{\mathbf{r}_i, \uparrow}, \hat{C}_{\mathbf{r}_i, \downarrow})$ denotes the spinor annihilation operators, and Δ_x and Δ_y denote the lattice vectors between two nearest-neighbored lattice sites along the x and y directions, respectively. The last term in the Hamiltonian (1) describes the coupling between the leads and the SO coupled region, $H_{s-l} = -\sum_p t_{ps} \sum_n (\hat{C}_{\mathbf{p}_n, \sigma}^\dagger \hat{C}_{\mathbf{r}_n, \sigma} + H.C.)$, where \mathbf{p}_n denotes a boundary lattice site in lead p connected directly to a boundary lattice site \mathbf{r}_n in the SO coupled region and t_{ps} the hopping parameter between lead p and the SO coupled region. It should be noticed that in general the TB Hamiltonian will also contain an on-site energy term, which is not explicitly shown above.

Now we consider the scattering of a conduction electron incident on a lead p by the SO coupled region. For conveniences, we will adopt a separate local coordinate frame in each lead, i.e., we will use a double coordinate index (x_p, y_p) to denote a lattice site in lead p , where $x_p = 1, 2, \dots, \infty$ (away from the border between the lead and the SO coupled region) and $y_p = 1, \dots, N_p$ (N_p is the width of lead p). In the local coordinate frame, the spatial wave function of a conduction electron incident on lead p will be given by $e^{-ik_m^p x_p} \chi_m^p(y_p)$, where k_m^p denotes the longitudinal wave vector and $\chi_m^p(y_p)$ the transverse spatial wave function and m the label of the transverse mode. The longitudinal wave vector will be determined by the following dispersion relation, $-2t \cos(k_m^p) + \varepsilon_m^p = E$, where ε_m^p is the eigen-energy of the m th transverse mode and E the energy of the incident electron. It should be noted that, due to the presence of SO coupling in the central scattering region, spin-flip processes (e.g., the spin-flip reflection) will be induced in the leads when a conduction electron is scattered or reflected by the central scattering region, even if the leads are ideal and nonmagnetic (which is the just case assumed in the present paper). Due to this fact, for a conduction electron incident from the m th transverse channel of lead p and with a *given* spin index σ , both the scattering wave function $|\psi^{pm\sigma}(\mathbf{r})\rangle$ in the central SO coupled region and the scattering wave function $|\psi^{pm\sigma}(\mathbf{x}_{p'})\rangle$ ($\mathbf{x}_{p'} \equiv (x_{p'}, y_{p'})$) in a lead p' will be inherently a superposition of a spin-up and a spin-down components,

$$|\psi^{pm\sigma}(\mathbf{r})\rangle = \sum_{\sigma'} \psi_{\sigma'}^{pm\sigma}(\mathbf{r}) \hat{C}_{\mathbf{r}_i, \sigma'}^\dagger |0\rangle, \quad (3a)$$

$$|\psi^{pm\sigma}(\mathbf{x}_{p'})\rangle = \sum_{\sigma'} \psi_{\sigma'}^{pm\sigma}(\mathbf{x}_{p'}) \hat{C}_{\mathbf{x}_{p'}, \sigma'}^\dagger |0\rangle, \quad (3b)$$

where $|0\rangle$ stands for the vacuum state. The spin-resolved components $\psi_{\sigma'}^{pm\sigma}(\mathbf{x}_{p'})$ of the scattering wave function in lead p' can be expressed in the following general form,

$$\begin{aligned} \psi_{\sigma'}^{pm\sigma}(\mathbf{x}_{p'}) &= \delta_{pp'} \delta_{\sigma\sigma'} e^{-ik_m^p x_p} \chi_m^p(y_p) \\ &+ \sum_{m' \in p'} \phi_{p'm'\sigma'}^{pm\sigma} e^{ik_{m'}^{p'} x_{p'}} \chi_{m'}^{p'}(y_{p'}), \end{aligned} \quad (4)$$

where $\phi_{p'm'\sigma'}^{pm\sigma}$ stands for the scattering amplitude from the $(m\sigma)$ channel of lead p (the *incident* mode) to the $(m'\sigma')$ channel of lead p' (the *out-going* mode). If $p' = p$, the second term on the right-hand side of Eq.(4) will denote actually the spin-resolved reflected waves in lead p and $\phi_{p'm'\sigma'}^{pm\sigma}$ denote the spin-flip ($\sigma \neq \sigma'$) and non-spin-flip ($\sigma = \sigma'$) reflection amplitudes. The scattering amplitudes $\phi_{p'm'\sigma'}^{pm\sigma}$ can be obtained by solving the Schrödinger equation for the entire structure. Since Eq.(4) is just a linear combination of all out-going modes with the same energy E in lead p' , the Schrödinger equation is satisfied automatically in lead p' except at those boundary lattice sites in lead p' connected directly to the SO coupled region. Due to the coupling between the

leads and the SO coupled region, the amplitudes of the wave function at these boundary lattice sites (which are determined by the scattering amplitudes $\phi_{qn\sigma'}^{pm\sigma}$) must be solved simultaneously with the wave function $\psi^{pm\sigma}(\mathbf{r})$ inside the SO coupled region. From the discrete version of the Schrödinger equation, one can show that the amplitudes of the wave function in the SO coupled region and at those boundary lattice sites of the external leads connected directly to the SO coupled region will satisfy the following coupled equations:

$$\begin{aligned} E\psi_{\sigma'}^{pm\sigma}(\mathbf{r}_s) &= \sum_{\mathbf{r}_s', \sigma''} H_{sys}(\mathbf{r}_s, \mathbf{r}_s', \sigma', \sigma'') \psi_{\sigma''}^{pm\sigma}(\mathbf{r}_s') \\ &- \sum_{p', y_{p'}} t_{p's} \delta_{\mathbf{r}_s, n_{p'} y_{p'}} \psi_{\sigma'}^{pm\sigma}(1, y_{p'}), \end{aligned} \quad (5a)$$

$$\begin{aligned} E\psi_{\sigma'}^{pm\sigma}(\mathbf{x}_{p'}) &= \sum_{\mathbf{x}_{p'}''} H_{p'}(\mathbf{x}_{p'}, \mathbf{x}_{p'}', \sigma', \sigma'') \psi_{\sigma''}^{pm\sigma}(\mathbf{x}_{p'}'') \\ &- \sum_{\mathbf{r}_s} t_{p's} \delta_{\mathbf{r}_s, n_{p'} y_{p'}} \psi_{\sigma'}^{pm\sigma}(\mathbf{r}_s), \end{aligned} \quad (5b)$$

where $(\mathbf{r}_s, \mathbf{r}_s')$ denote two nearest-neighbored lattice sites in the SO coupled region and $(\mathbf{x}_{p'}', \mathbf{x}_{p'}'')$ two nearest-neighbored boundary lattice sites in lead p' . (For simplicity of notation, in the subscript of the Kronecker δ -function we have used simply a symbol $n_{p'} y_{p'}$ to denote a boundary lattice site in the SO coupled region which is connected directly to a boundary lattice site $\mathbf{x}_{p'}' = (1, y_{p'})$ in lead p' .) The matrix elements $H_{sys}(\mathbf{r}_s, \mathbf{r}_s', \sigma', \sigma'')$ and $H_{p'}(\mathbf{x}_{p'}', \mathbf{x}_{p'}'', \sigma', \sigma'')$ can be written down directly from the Hamiltonian (1). Eq.(5a) and (5b) are the match conditions of the scattering wave function on the borders between the SO coupled region and the leads, from which both the scattering wave function in the entire structure and all scattering amplitudes can be obtained simultaneously. Some details of the derivations are given in the appendix.

III. SOME RIGOROUS PROPERTIES OF EQUILIBRIUM STATES

A controversial issue encountered in the study of spin-polarized transports in intrinsically SO coupled systems is that whether there can exist nonvanishing *equilibrium background spin currents* in such systems. Recently Rashba pointed out that, in a bulk two-dimensional electron gas (2DEG) with Rashba SO coupling, a finite equilibrium background spin current could be obtained if the conventional definition of spin current is applied to such systems.⁸ If this equilibrium background spin current does exist, it would imply that nonvanishing equilibrium spin polarizations should also exist near the edges of such a system due to the flow of the equilibrium background spin current. It was argued in Ref.[8] that such equilibrium background spin currents are an artefact caused by

the improper use of the conventional definition of spin current to an intrinsically SO coupled system, i.e., the conventional definition of spin current cannot be applied in the presence of intrinsic SO coupling. Since it seemed that no consensus had been arrived on whether there is a uniquely correct definition for spin current in a SO coupled system^{28,29,30,31}, it would be meaningful if this controversial issue can be investigated in a somewhat different way. In this section we will use the scattering wave function approach introduced in section II to investigate whether there can exist nonvanishing equilibrium background spin currents and/or nonvanishing equilibrium spin polarizations in a multi-terminal mesoscopic SO coupled system. Based on some simple but rigorous arguments, we will show that no finite equilibrium spin polarizations and/or finite equilibrium terminal spin currents can exist in a multi-terminal mesoscopic SO coupled system.

A. Absence of equilibrium spin polarizations

In the tight-binding representation, the operator for the local spin density at a lattice site i reads

$$\hat{S}(i) = \frac{\hbar}{2} \sum_{\alpha\beta} \hat{C}_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \hat{C}_{i\beta}. \quad (6)$$

(For simplicity of notation, from now on we will use simply a symbol i to denote a lattice site in the entire structure, i.e., both in the SO coupled region and in the external leads.) Under time reversal transformation, the local spin density operator will transform as $\hat{S}(i) \rightarrow \hat{T} \hat{S}(i) \hat{T}^{-1} = -\hat{S}(i)$ and the spin operator transform as $\vec{\sigma}_{\alpha\beta} \rightarrow \hat{T} \vec{\sigma}_{\alpha\beta} \hat{T}^{-1} = (-1)^{\alpha+\beta} \vec{\sigma}_{\bar{\alpha}\bar{\beta}}^* = -\vec{\sigma}_{\alpha\beta}$, where $\bar{\alpha} \equiv -\alpha$, $\bar{\beta} \equiv -\beta$, $\hat{T} \equiv i\sigma_y \hat{K}$ denotes the time-reversal transformation operator and \hat{K} the conjugate operator.

Within the framework of the standard Landauer-Büttiker's formalism, any physical quantities of a mesoscopic system are contributed to by all scattering states of conduction electrons incident from all contacts. These scattering states constitute an ensemble which can be specified by a chemical potential μ_p for each contact through their separate Fermi distribution function $f(E, \mu_p)$. For the problems discussed in the present paper, this ensemble consists of all scattering states described by the scattering wave functions $\{\psi^{pm\sigma}\}$ given by Eqs.(3-4). In order that there is only one particle feeding into each incident channel³³, when we use this ensemble of scattering wave functions to calculate the expectation value of an operator, one should first normalize the scattering wave function $\psi^{pm\sigma}$ by a factor of $1/\sqrt{L}$ ($L \rightarrow \infty$ is the length of lead p), corresponding to that one changes the incident wave functions from $e^{ik_m^p x_p}$ to $e^{ik_m^p x_p}/\sqrt{L}$ ³³. By use of the ensemble of the normalized scattering wave functions $\{\psi^{pm\sigma}\}$ and noticing that the density of states (DOS) for the m th transverse mode of lead p is given by $\frac{L}{2\pi} \frac{dk}{dE} = \frac{L}{2\pi\hbar v_{pm}}$, where

$v_{pm} = \frac{2t_p}{\hbar} \sin(k_m^p)$ is the longitudinal velocity of the m th transverse mode of lead p , then one can see that the local spin density at a lattice site i (either in the SO coupled region or in the external leads) will be given by

$$\begin{aligned} \langle \hat{S}(i) \rangle &= \sum_{pm\sigma} \int \frac{dE}{2\pi} f(E, \mu_p) \frac{1}{\hbar v_{pm}} \\ &\times \sum_{\alpha\beta} [\psi_\alpha^{pm\sigma*}(i) (\frac{\hbar}{2} \vec{\sigma})_{\alpha\beta} \psi_\beta^{pm\sigma}(i) + H.C.], \quad (7) \end{aligned}$$

where $\psi^{pm\sigma}$ is the scattering wave function corresponding to an incident electron from the $(m\sigma)$ channel of lead p with a given energy E . *This formula is valid both in the equilibrium and in the nonequilibrium states.* If the system is in an equilibrium state, the chemical potential μ_p will be independent of the lead label, i.e., $\mu_p \equiv \mu$ and $f(E, \mu_p) \equiv f(E, \mu)$. Then in Eq.(7) the summation $\sum_{pm\sigma} [\dots]$ can be performed first before carrying out the integration over energy E , and the result for this summation can be expressed as

$$\begin{aligned} &\sum_{\alpha\beta} \sum_{pm\sigma} [\psi_\alpha^{pm\sigma*}(i) (\frac{\hbar}{2} \vec{\sigma})_{\alpha\beta} \psi_\beta^{pm\sigma}(i) / \hbar v_{pm} + H.C.] \\ &= \sum_{\alpha\beta} [A_{\beta\alpha}(i, i; E) (\frac{\hbar}{2} \vec{\sigma})_{\alpha\beta} + H.C.] \\ &= i \sum_{\alpha\beta} \{ [G^R(E) - G^A(E)]_{i\beta, i\alpha} (\frac{\hbar}{2} \vec{\sigma})_{\alpha\beta} - H.C. \}. \quad (8) \end{aligned}$$

Here $G^{R,A}(E) = [E\mathbf{I} - \hat{H} \pm i0^\pm]^{-1}$ is just the retarded and advanced Green's functions for the system, whose explicit spin-resolved matrix forms are given by $G_{i\alpha, j\beta}^{R,A}(E) = \sum_{p'm'\sigma'} \psi_\alpha^{p'm'\sigma'*}(i) \psi_\beta^{p'm'\sigma'}(j) / [E - E' \pm i0^\pm]^{-1}$ (E' is the incident energy corresponding to a scattering wave function $\psi^{p'm'\sigma'}$); and $A_{\beta\alpha}(j, i; E) \equiv \sum_{p'm'\sigma'} \psi_\alpha^{p'm'\sigma'*}(i) \psi_\beta^{p'm'\sigma'}(j) / \hbar v_{p'm'} = i[G^R(E) - G^A(E)]_{j\beta, i\alpha}$ is the spin-resolved spectral function. If the total Hamiltonian \hat{H} for the entire system is time-reversal invariant, the retarded and advanced Green's functions can be related by the time reversal transformation as

$$G_{i\alpha, j\beta}^A = (\hat{T} G^R \hat{T}^{-1})_{i\alpha, j\beta} = (-1)^{\alpha+\beta} G_{i\bar{\alpha}, j\bar{\beta}}^{R*} \quad (9)$$

Combining Eq.(8) and Eq.(9) and taking into account the fact that $\hat{T} \vec{\sigma}_{\alpha\beta} \hat{T}^{-1} = (-1)^{\alpha+\beta} \vec{\sigma}_{\bar{\alpha}\bar{\beta}}^* = -\vec{\sigma}_{\alpha\beta}$, one gets immediately that the right-hand side of Eq.(8) should vanish exactly, thus the spin density given by Eq.(7) vanishes exactly in the equilibrium state, suggesting that no finite equilibrium spin polarizations can survive at any lattice site i in the entire structure (both in the SO coupled region and in the leads). It should be noted that in arriving at this conclusion we have only made use of the assumption that the total Hamiltonian \hat{H} for the entire system is time-reversal invariant (which should be the

case in the absence of magnetic fields) and did not involve the actual form of the SO coupling in the system, so it is a much general conclusion.

B. Absence of equilibrium terminal spin currents

In this subsection we discuss whether there can exist nonvanishing equilibrium terminal spin currents in a multi-terminal mesoscopic SO coupled system. Since we have assumed that the leads are ideal and nonmagnetic (i.e., described by a simple Hamiltonian $\hat{H}_p = -t_p \sum_{\langle \mathbf{p}_i, \mathbf{p}_j \rangle \sigma} (\hat{C}_{\mathbf{p}_i \sigma}^\dagger \hat{C}_{\mathbf{p}_j \sigma} + H.C.)$), the conventional definitions of charge and spin currents can be well applied in the leads without ambiguities. According to the conventional definitions and in the lattice representation, the charge current and spin current (with spin parallel to the α axis³⁴) flowing from a lattice site \mathbf{p}_i to a nearest-neighbored lattice site \mathbf{p}_j in lead p can be given by the the corresponding particle density current as following,

$$\hat{I}_{p, \mathbf{p}_i \rightarrow \mathbf{p}_j} = e[\hat{J}_{\mathbf{p}_i \rightarrow \mathbf{p}_j}^+ + \hat{J}_{\mathbf{p}_i \rightarrow \mathbf{p}_j}^-], \quad (10a)$$

$$\hat{I}_{p, \mathbf{p}_i \rightarrow \mathbf{p}_j}^\alpha = \frac{\hbar}{2}[\hat{J}_{\mathbf{p}_i \rightarrow \mathbf{p}_j}^+ - \hat{J}_{\mathbf{p}_i \rightarrow \mathbf{p}_j}^-], \quad (10b)$$

where $\hat{I}_{p, \mathbf{p}_i \rightarrow \mathbf{p}_j}$ denotes the charge current operator and $\hat{I}_{p, \mathbf{p}_i \rightarrow \mathbf{p}_j}^\alpha$ the spin current operator (with spin parallel to the α axis) and $\hat{J}_{\mathbf{p}_i \rightarrow \mathbf{p}_j}^\sigma$ the spin-resolved particle density current operator and $\sigma = \pm$ denotes the spin-up and spin-down states with respect to the α axis. From the Heisenberg equation of motion for the on-site particle density: $\frac{d}{dt} \hat{N}_{\mathbf{p}_i} = \frac{1}{i\hbar} [\hat{N}_{\mathbf{p}_i}, \hat{H}_p]$, where $\hat{N}_{\mathbf{p}_i} = \hat{C}_{\mathbf{p}_i \sigma}^\dagger \hat{C}_{\mathbf{p}_i \sigma}$ is the on-site particle density operator in lead p , one can show easily that the spin-resolved particle density current flowing from a lattice site \mathbf{p}_i to a nearest-neighbored lattice \mathbf{p}_j in lead p will be given by

$$\hat{J}_{\mathbf{p}_i \rightarrow \mathbf{p}_j}^\sigma = \frac{it_p}{\hbar} (\hat{C}_{\mathbf{p}_j \sigma}^\dagger \hat{C}_{\mathbf{p}_i \sigma} - \hat{C}_{\mathbf{p}_i \sigma}^\dagger \hat{C}_{\mathbf{p}_j \sigma}). \quad (11)$$

Now we calculate the terminal charge and spin currents flowing along the *longitudinal* direction of a lead q . Firstly we consider the contribution of an incident electron from the $(m\sigma)$ channel of lead p to the longitudinal charge and spin currents (with spin parallel to the α axis) flowing through a transverse cross-section (saying, e.g., the cross-section at $x = x_q$) of lead q , which by definition will be given by

$$\begin{aligned} \langle \hat{I}_q \rangle_{pm\sigma} &= \frac{1}{L} \sum_{y_q} \langle \psi^{pm\sigma}(x_q + 1, y_q) | \\ &\quad \times \hat{I}_{q, (x_q, y_q) \rightarrow (x_q + 1, y_q)} | \psi^{pm\sigma}(x_q, y_q) \rangle \\ &= \frac{e}{L} \left\{ \sum_{n, \sigma'} v_{qn} |\phi_{qn\sigma'}^{pm\sigma}|^2 - v_{pm} \delta_{pq} \right\}, \quad (12a) \end{aligned}$$

$$\begin{aligned} \langle \hat{I}_q^\alpha \rangle_{pm\sigma} &= \frac{1}{L} \sum_{y_q} \langle \psi^{pm\sigma}(x_q + 1, y_q) | \\ &\quad \times \hat{I}_{q, (x_q, y_q) \rightarrow (x_q + 1, y_q)}^\alpha | \psi^{pm\sigma}(x_q, y_q) \rangle \\ &= \frac{\hbar}{4\pi L} \left\{ \sum_n v_{qn} [|\phi_{qn+}^{pm\sigma}|^2 - |\phi_{qn-}^{pm\sigma}|^2] - \sigma v_{pm} \delta_{pq} \right\}, \quad (12b) \end{aligned}$$

where (x_q, y_q) and $(x_q + 1, y_q)$ denote two nearest-neighbored lattice sites along the longitudinal direction of lead q , $\frac{1}{\sqrt{L}} |\psi^{pm\sigma}(x_q, y_q)\rangle$ denotes the *normalized* scattering wave function in lead q corresponding to the incident electron from the $(m\sigma)$ channel of lead p (which is given by Eqs.(3-4)), and $v_{qn} = \frac{2t_p}{\hbar} \sin(k_n^q)$ denotes the longitudinal velocity of the n th transverse mode in lead q and $v_{pm} = \frac{2t_p}{\hbar} \sin(k_m^p)$ the longitudinal velocity of the m th transverse mode in lead p . The summation over the transverse coordinate y_q runs over from 1 to N_q (N_q is the width of lead q)³⁵, and the following orthogonality relations for transverse modes in lead q have been applied in obtaining the last lines of Eq.(12a) and (12b): $\sum_{y_q} \chi_m^q(y_q) \chi_n^q(y_q) = \delta_{mn}$. It should be noted that, if $p = q$, the results given by Eq.(12a) and (12b) will denote actually the contribution of an incident electron from lead q to the charge and spin currents flowing in the same lead and $\phi_{qn\sigma'}^{pm\sigma}$ ($\sigma' = \pm$) denote actually the spin-flip ($\sigma \neq \sigma'$) and non-spin-flip ($\sigma = \sigma'$) reflection amplitudes. (See the explanations given to Eqs.(3-4) in section II). In such cases, the results given by Eq.(12a) and (12b) can be expressed as the subtraction of the contributions due to the incident wave (i.e., the terms proportional to δ_{pq} in Eq.(12a) and (12b)) and the contributions due to the spin-flip and non-spin-flip reflected waves.

The total terminal charge current I_q and the total terminal spin current I_q^α flowing in lead q will be obtained by summing the contributions of all incident electrons from all leads with the corresponding density of states (see the explanations given above Eq.(7)). Then we get that

$$\begin{aligned} I_q &= \frac{e}{h} \sum_{pm\sigma} \int dE f(E, \mu_p) \left[\sum_{n, \sigma'} |\phi_{qn\sigma'}^{pm\sigma}|^2 \frac{v_{qn}}{v_{pm}} - \delta_{pq} \right] \\ &= \frac{e}{h} \sum_{p\sigma\sigma'} \int dE f(E, \mu_p) [T_{q\sigma'}^{p\sigma}(E) - \delta_{pq} \delta_{\sigma\sigma'} N_q(E)] \\ &= \frac{e}{h} \sum_{p\sigma\sigma'} \int dE [f(E, \mu_p) T_{q\sigma'}^{p\sigma}(E) - f(E, \mu_q) T_{p\sigma'}^{q\sigma}(E)], \quad (13a) \end{aligned}$$

$$\begin{aligned} I_q^\alpha &= \sum_{pm\sigma} \int dE f(E, \mu_p) \left[\sum_n \frac{v_{qn}}{4\pi v_{pm}} (|\phi_{qn+}^{pm\sigma}|^2 - |\phi_{qn-}^{pm\sigma}|^2) \right] \\ &= \frac{1}{4\pi} \sum_{p\sigma} \int dE f(E, \mu_p) [T_{q+}^{p\sigma}(E) - T_{q-}^{p\sigma}(E)], \quad (13b) \end{aligned}$$

where $T_{q\sigma'}^{p\sigma}(E) \equiv \sum_{m,n} \left| \phi_{qn\sigma'}^{pm\sigma} \right|^2 \frac{v_{qn}}{v_{pm}}$ denotes (by definition) the transmission probability from lead p with spin σ to lead q with spin σ' (see Ref.[33] and also the explanations given in the appendix A), and in obtaining the last line of Eq.(13a) the following relation has been applied³³:

$$\sum_{p\sigma} T_{q\sigma'}^{p\sigma}(E) = \sum_{p\sigma} T_{q\sigma'}^{p\sigma}(E) = N_q(E), \quad (14)$$

where $N_q(E)$ is the total number of conducting transverse modes in lead q corresponding to a given energy E . As was discussed in detail in Ref.[33], this relation follows directly from the unitarity of the S-matrix, which is essential for the particle number conservation.

Eq.(13a) is just the usual Landauer-Büttiker formula for terminal charge currents in a multi-terminal mesoscopic system³³. The second line in Eq.(13a) indicates clearly that the terminal charge current flowing in lead q can be expressed as the subtraction of the contributions due to all incident modes (corresponding to the terms proportional to δ_{pq}) and the contributions due to all outgoing modes (corresponding to the terms proportional to $T_{q\sigma'}^{p\sigma}$), which include both the transmitted waves from other leads ($p \neq q$) and the reflected waves in lead q ($p = q$), noticing that $T_{q\sigma'}^{p\sigma}$ denotes actually the spin-flip or non-spin-flip reflection probabilities from lead q to lead q if $p = q$. Eq.(13b) is somewhat different from Eq.(13a) in appearance, but the terminal spin current given by Eq.(13b) can still be divided into two different kinds of contributions, namely the contributions due to the transmitted waves from other leads (corresponding to those terms with $p \neq q$ in the summation $\sum_{p\sigma} [\dots]$) and the contributions due to the spin-flip and non-spin-flip reflected waves in lead q (corresponding to those terms with $p = q$ in the summation $\sum_{p\sigma} [\dots]$).³⁶

Eq.(13a) and (13b) are valid both in the equilibrium and in the nonequilibrium states. In the equilibrium state, since $\mu_p \equiv \mu$ (independent of the lead label) and $f(E, \mu_p) \equiv f(E, \mu)$, in Eq.(13a) and (13b) the summation $\sum_{p\sigma} [\dots]$ can be performed first before carrying out the energy integration and we get that

$$I_q = \frac{e}{h} \int dE f(E, \mu) \sum_{p\sigma\sigma'} [T_{q\sigma'}^{p\sigma}(E) - T_{p\sigma'}^{q\sigma}(E)], \quad (15a)$$

$$I_q^\alpha = \frac{1}{4\pi} \int dE f(E, \mu) \sum_{p\sigma} [T_{q+}^{p\sigma}(E) - T_{q-}^{p\sigma}(E)]. \quad (15b)$$

Then by use of Eq.(14) one can see clearly that both terminal charge currents and terminal spin currents will vanish exactly in the equilibrium state. It should be stressed that in arriving at this conclusion we did not involve the controversial issue of what is the correct definition of spin current in the central SO coupled region at all, so those ambiguities that might be caused by the use of an improper definition of spin current to the SO coupled region have been eliminated in our derivations. Though we

cannot prove that the spin current also vanishes exactly inside the SO coupled region based on the approach applied above, however, for a mesoscopic system only the terminal (charge or spin) currents are the real useful quantities from the *practical* point of view (i.e., one need to add external contacts to induct the charge or spin currents out of a mesoscopic sample). We note that a similar conclusion as was obtained above has also been derived in Ref.[37] based on some somewhat different arguments. Compared with the derivations given in Ref.[37], the arguments given above seem to be more simple and more transparent in principle. It also should be noted that, based on a similar Landauer-Büttiker formalism, it was argued in Ref.[38] that the equilibrium terminal spin currents should indeed take place in a three terminal system with spin-orbit coupling³⁸, in contradiction to the conclusion obtained in the present paper and in Ref.[37]. To our understandings, this contradiction was caused by the fact that the contributions due to the spin-flip and non-spin-flip reflections in the leads (induced by the scatterings from the central SO coupled region) was neglected in the calculations of terminal spin currents performed in Ref.[38]. In contrast, in the calculations of terminal spin currents performed in the present paper, the contributions due to the spin-flip and non-spin-flip reflections in the leads induced by the scatterings from the central SO coupled region have been treated in an accurate and strict way, assuming that the leads are ideal and nonmagnetic.

IV. SOME SYMMETRY PROPERTIES OF NONEQUILIBRIUM SPIN CURRENTS AND SPIN POLARIZATIONS IN TWO-TERMINAL MESOSCOPIC SO COUPLED SYSTEMS

When a multi-terminal mesoscopic SO coupled system is driven into a nonequilibrium state (i.e., there is charge current flow between different leads), nonequilibrium spin polarizations and/or terminal spin currents may be induced by the charge current flow. In this section we discuss some symmetry properties of such nonequilibrium spin currents and spin polarizations. For clarity, we take a typical two-terminal mesoscopic structure as shown in Fig.?? as the example, where a ballistic two-dimensional electron gas (2DEG) with Rashba and/or k-linear Dresselhaus SO coupling is attached to two ideal leads.

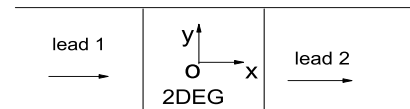


FIG. 2: Schematic geometry of a two-terminal mesoscopic SO coupled system.

A. Non-antisymmetric lateral edge spin accumulations

The study of nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of a two-dimensional electron gas with intrinsic SO coupling is of great theoretical interest because of its close relations with the intrinsic spin Hall effect in such systems. It was generally believed that the principal observable signature of the intrinsic spin Hall effect in a SO coupled system is that, when a longitudinal charge current circulates through such a system with a thin strip geometry, *antisymmetric* lateral edge spin accumulation (polarized perpendicular to the 2DEG plane) will be induced at the two lateral edges of the strip due to the flow of the transverse spin Hall current. Several recent numerical calculations had demonstrated that a longitudinal charge current circulating through a thin strip of a ballistic two-dimensional electron gas with Rashba SO coupling does can lead to the generation of antisymmetric edge spin accumulation at the two lateral edges of the strip, and the antisymmetric character of the transverse spatial distribution of the lateral edge spin accumulation (i.e., $\langle S_z(x, y) \rangle = -\langle S_z(x, -y) \rangle$) had been argued to be a strong support of the existence of intrinsic spin Hall effect in such mesoscopic SO coupled systems¹⁵. Here we discuss this issue from a different point of view. We will show that, when a longitudinal charge current circulates through a thin strip of a ballistic two-dimensional electron gas with both Rashba and Dresselhaus SO coupling, the transverse spatial distribution of the induced nonequilibrium lateral edge spin accumulation (polarized perpendicular to the 2DEG plane) are in general non-antisymmetric. The non-antisymmetric character of the lateral edge spin accumulation contradicts seriously with the usual physical pictures of spin Hall effect, though according to some theoretical predictions, intrinsic spin Hall effect should also survive in the presence of both Rashba and Dresselhaus SO coupling¹². The non-antisymmetric character of the lateral edge spin accumulation implies that, in addition to the intrinsic spin Hall effect, there may exist some other physical reasons that might also lead to the generation of nonequilibrium lateral edge spin accumulation in a SO coupled system (with a thin strip geometry) when a longitudinal charge current circulates through it.^{23,24}

Firstly let us look at what symmetry relations can be obtained for the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current based on the symmetry analysis of the Hamiltonian of the system under study (sketched in Fig.2). If only Rashba (or only Dresselhaus) SO coupling presents, based on the symmetry properties of the Hamiltonian of the structure under study, one can show rigorous that the nonequilibrium lateral edge spin accumulation does should be antisymmetric at the two lateral edges. Let us consider first the case in which only Rashba SO coupling presents (i.e., the Dresselhaus SO coupling strength is zero).

If only Rashba SO coupling presents, from Eq.(2a) and Fig.2 one can see that the Hamiltonian of the entire structure is invariant under the combined transformation of the real space reflection $y \Rightarrow -y$ and the spin space rotation around the S_y axis (with an angle π). From this invariance one can get immediately that $\langle S_z(x, y) \rangle_I = -\langle S_z(x, -y) \rangle_I$, where $\langle S_z \rangle_I$ denotes the nonequilibrium spin density induced by a longitudinal charge current flowing from lead 1 to lead 2. It is interesting to note that this antisymmetric relation can be deduced directly from the symmetry of the structure under study but without need to resort to the concept of spin Hall effect at all. Next, let us consider the case in which only Dresselhaus SO coupling presents (i.e., the Rashba SO coupling strength is zero). If only Dresselhaus SO coupling presents, then from Eq.(2b) and Fig.2 one can see that the Hamiltonian of the entire structure is invariant under the combined transformation of the real space reflection $y \Rightarrow -y$ and the spin space rotation around the S_x axis (with an angle π). From this invariance one also gets immediately that $\langle S_z(x, y) \rangle_I = -\langle S_z(x, -y) \rangle_I$, i.e., the nonequilibrium lateral edge spin accumulation still should be antisymmetric at the two lateral edges if only Dresselhaus SO coupling presents.

If both Rashba and Dresselhaus SO couplings are present, then from Eqs.(2a-2b) and Fig.2 one can see that the total Hamiltonian of the entire structure is invariant under the combined transformation of the real space center inversion $\mathbf{r} \Rightarrow -\mathbf{r}$ and the spin space rotation around the S_z axis (with an angle π). From this invariance one can get that $\langle S_z(x, y) \rangle_I = \langle S_z(-x, -y) \rangle_{-I}$, where $\langle S_z \rangle_{-I}$ denotes the nonequilibrium spin density induced by a longitudinal charge current flowing from lead 2 to lead 1. On the other hand, from Eq.(7) one can see that in the linear response regime one has

$$\langle S_z(x, y) \rangle_I = -\langle S_z(x, y) \rangle_{-I}. \quad (16)$$

Combining the two relations $\langle S_z(x, y) \rangle_I = \langle S_z(-x, -y) \rangle_{-I}$ and $\langle S_z(x, y) \rangle_I = -\langle S_z(x, y) \rangle_{-I}$, then the following symmetry relation can be obtained for the nonequilibrium spin accumulation induced by a longitudinal charge current flowing from lead 1 to lead 2: $\langle S_z(x, y) \rangle_I = -\langle S_z(-x, -y) \rangle_I$. This symmetry relation implies that the transverse spatial distribution of the nonequilibrium lateral edge spin accumulation will be antisymmetric in the center cross-section of the strip, i.e., $\langle S_z(0, y) \rangle_I = -\langle S_z(0, -y) \rangle_I$. Due to the existence of this symmetry relation, one can deduce that in an *infinite* strip (i.e., the length of the strip tends to infinity and hence the effects of the contacted leads can be neglected), the transverse spatial distribution of the nonequilibrium lateral edge spin accumulation will still be antisymmetric (i.e., $\langle S_z(x, y) \rangle_I = -\langle S_z(x, -y) \rangle_I$ for all x) in the presence of both Rashba and Dresselhaus SO coupling. However, unlike the case in which only Rashba (or only Dresselhaus) SO coupling presents, for a thin strip of finite length, in the presence of both Rashba and Dresselhaus SO coupling, one cannot de-

duce a general antisymmetric relation for the transverse spatial distribution of the nonequilibrium lateral edge spin accumulation based on the symmetry analysis of the total Hamiltonian of the entire structure under study. From the theoretical points of view, this is due to the fact that, in the presence of both Rashba and Dresselhaus SO coupling, the total Hamiltonian of the entire structure under study (i.e., a thin strip of finite length contacted to two ideal leads) is no longer invariant under the combined transformation of the real space reflection $y \Rightarrow -y$ and the spin space rotation around the S_y (or S_x) axis with an angle π . Indeed, as will be shown below, this non-antisymmetric character can be verified by detailed numerical calculations.

One particular interesting case is that the Rashba and the Dresselhaus SO coupling strengths are equal (i.e., $t_R = t_D$ or $t_R = -t_D$). In this particular case, the total Hamiltonian is invariant under the following unitary transformation in spin space (while the real space coordinate \mathbf{r} remain unchanged): $\hat{U}_+ \hat{H} \hat{U}_+ = \hat{H}$ (if $t_R = t_D$) or $\hat{U}_- \hat{H} \hat{U}_- = \hat{H}$ (if $t_R = -t_D$), where $\hat{U}_+ = (\hat{\sigma}_x + \hat{\sigma}_y)/\sqrt{2}$ and $\hat{U}_- = (\hat{\sigma}_x - \hat{\sigma}_y)/\sqrt{2}$. Under this unitary transformation, the spin operators will transform as following: $\hat{\sigma}_z \rightarrow -\hat{\sigma}_z$, $\hat{\sigma}_x \rightleftharpoons \hat{\sigma}_y$ (if $t_R = t_D$) or $\hat{\sigma}_x \rightleftharpoons -\hat{\sigma}_y$ (if $t_R = -t_D$). Since the real space coordinate \mathbf{r} remain unchanged under this symmetry manipulation, from the above symmetry properties in spin space one gets immediately that $\langle S_z(x, y) \rangle_I = -\langle S_z(x, y) \rangle_I$, suggesting that $\langle S_z(x, y) \rangle_I$ should vanish everywhere if the Rashba and the Dresselhaus SO coupling strengths are equal. This conclusion is in exact agreement with the corresponding numerical results obtained based on the scattering wave approach introduced in section II-III, which shows that $\langle S_z(x, y) \rangle_I$ does vanish everywhere in the particular case of $t_R = t_D$ or $t_R = -t_D$.

To show more explicitly the non-antisymmetric character of the lateral edge spin accumulation induced by a longitudinal charge current in a 2DEG strip of finite length with both Rashba and Dresselhaus SO coupling, in Fig.3 we plotted a typical pattern of the two-dimensional spatial distribution of the nonequilibrium spin density $\langle S_z \rangle$ in the strip obtained by numerical calculations with the scattering wave function approach introduced in Sec.II-III. In our numerical calculations we take the typical values of the electron effective mass $m = 0.04m_e$, the lattice constant $a = 3nm$, and the 2DEG strip contains 120×40 lattice sizes. The chemical potentials in the two leads are set by fixing the longitudinal charge current to flow from lead 1 to lead 2 as shown in Fig.2 and fixing the longitudinal charge current density to $100\mu A/1.5\mu m$ (as reported in Ref.[26]). From Fig.3 one can see clearly that the transverse spatial distribution of the nonequilibrium spin density $\langle S_z \rangle$ in the strip is non-antisymmetric in general (i.e., $\langle S_z(x, y) \rangle_I \neq -\langle S_z(x, -y) \rangle_I$ for general x), except in the center cross-section (i.e., $x = 0$) of the strip. The non-antisymmetric character of the transverse spatial distribution of the nonequilibrium spin density $\langle S_z \rangle$ can be more clearly seen from Fig.4(a), where we plotted

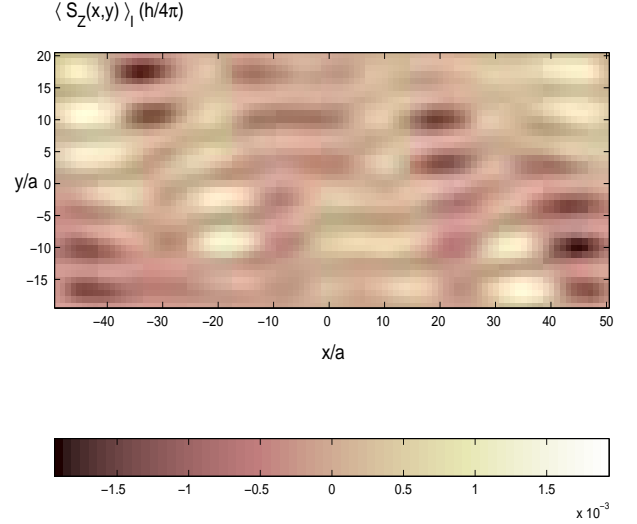


FIG. 3: (Color online) A typical pattern of the two-dimensional spatial distribution of the current induced nonequilibrium spin density $\langle S_z \rangle$ in a two-terminal structure (sketched in Fig.2) in the presence of both Rashba and Dresselhaus SO coupling. The Rashba and Dresselhaus SO coupling strength is set to $t_R/t = 0.08$ and $t_D/t = 0.02$.

several typical patterns of the profiles of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z \rangle$ in a cross-section of the strip at $x \neq 0$. (For comparison, the corresponding results obtained in the case that only Rashba or only Dresselhaus SO coupling presents were also plotted in Fig.4(b)). The three typical patterns shown in Fig.4(a) are obtained by fixing the Dresselhaus SO coupling strength to $t_D = 0.02t$ (t is the spin-independent hopping parameter) and varying the Rashba SO coupling strength t_R . From Fig.4(a) one can see clearly that, the transverse spatial distribution of the nonequilibrium spin density $\langle S_z(x, y) \rangle_I$ can have either the same signs or opposite signs at the two lateral edges of the strip, depending on the ratios of t_R/t_D . Even in the case that $\langle S_z \rangle$ has opposite signs at the two lateral edges, the transverse spatial distributions of $\langle S_z \rangle$ may still not be antisymmetric (i.e., $\langle S_z(x, y) \rangle \neq -\langle S_z(x, -y) \rangle$), contradicting significantly with the usual physical pictures of spin Hall effect. The non-antisymmetric character of the lateral edge spin accumulation suggests that some cautions may need to be taken when attributing the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of a SO coupled system to a spin Hall effect, especially in the mesoscopic regime.

The results shown in Figs.3-4 are obtained in the absence of impurity scatterings. One can show that the symmetry properties shown in Fig.3-4 are robust against spinless weak impurity scatterings. To model spinless weak disorder scatterings, we assume that the on-site energy at lattice sites in the 2DEG strip are randomly distributed in a narrow energy region $[-W, W]$, where

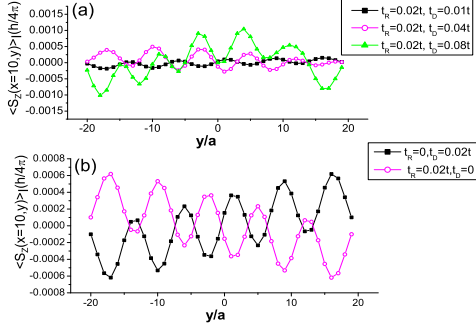


FIG. 4: (Color online)(a) Some typical profiles of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z \rangle$ induced by a longitudinal charge current in a 2DEG strip with both Rashba and Dresselhaus SO couplings, which shows clearly that the transverse spatial distributions of $\langle S_z \rangle$ are non-antisymmetric in general at both edges of the strip in the presence of both Rashba and Dresselhaus SO coupling. $\langle S_z \rangle$ vanishes everywhere in the particular case of $t_R = t_D$ or $t_R = -t_D$ (not shown explicitly in the figure). (b) The corresponding results obtained in the case that only Rashba or only Dresselhaus SO coupling presents, which shows clearly that the transverse spatial distributions of $\langle S_z \rangle$ are antisymmetric at both edges of the strip if only Rashba or only Dresselhaus SO coupling presents.

W is the amplitude of the on-site energy fluctuations characterizing the disorder strength. (In the absence of disorder scatterings, the on-site energy at each lattice site can be set simply to zero.) We calculate the spin density for a number of random impurity configurations and then do impurity average. In Fig.5 we show the variations of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z \rangle$ in a cross-section of the strip as the disorder strength increases, from which one can see that the symmetry properties of the transverse spatial distributions of the nonequilibrium spin density are robust against spinless weak disorder scatterings.

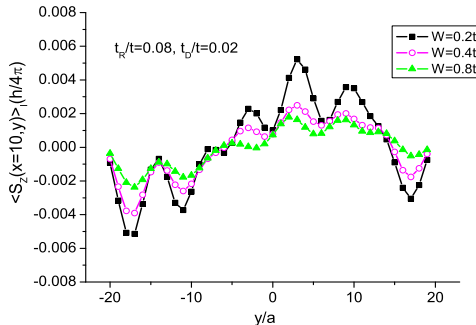


FIG. 5: (Color online)The profiles of the transverse spatial distributions of the nonequilibrium spin density $\langle S_z \rangle$ in the presence of disorder. We have done impurity average over 1000 random impurity configurations for each case.

B. Non-conservative terminal spin currents

As has been mentioned in the introduction section, a much controversial issue related to the study of spin-polarized transports in SO coupled systems is that whether spin currents should be a conserved quantity and what is the correct definition of spin currents in such systems. It was now well established that, in the presence of SO coupling, spin current calculated based on the conventional definition is not a conserved quantity, and in order to make spin current a conserved quantity in the presence of SO coupling, significant modifications to the conventional definition will be needed.^{28,30} Nevertheless, it seemed that no consensus had been arrived on whether spin current should be a conserved quantity in a SO coupled system or whether there is a uniquely correct definition for spin current in such a system.^{28,29,30,31} Below we will discuss this controversial issue from a different point of view, i.e., we do not consider the problem that what is the correct definition of spin current in a SO coupled system but focus our discussion on the question that whether the terminal spin currents in a multi-terminal mesoscopic SO coupled system are conservative. As mentioned earlier, for a mesoscopic SO coupled system, only the terminal spin currents are the real useful quantities. By use of the two-probe mesoscopic structure shown in Fig.2 as the example, we will show explicitly that the terminal spin currents in a multi-terminal mesoscopic SO coupled system are non-conservative in general, i.e., the total spin currents flowing into the SO coupled region are not equal to the total spin currents flowing out of the same region. To illustrate this point clearly, we take a two-terminal mesoscopic system with both Rashba and Dresselhaus SO coupling as the example. In Fig.6(a) and (b) we plotted the terminal spin currents I_1^z and I_2^z in the two leads (with spin parallel to the z axis) and the terminal spin currents I_1^y and I_2^y in the two leads (with spin parallel to the y axis) as a function of the Rashba SO coupling strength t_R , respectively. In our calculations we fix the Dresselhaus SO coupling strength to $t_D = 0.02t$ and fix the longitudinal charge current density to $100\mu A/1.5\mu m$. The lattice constant $a = 3nm$ and the lattice size of the 2DEG strip is taken to be 100×40 . The positive direction of the spin current flow is defined to be from lead 1 to lead 2. From Fig.6(a) one can see that the terminal spin currents I_1^z and I_2^z in the two leads have the same signs, which means that the terminal spin currents with spin parallel to the z axis will flow from lead 1 into the SO coupled region and then flow out of the SO coupled region into lead 2, similar to the usual charge current transport. From Fig.6(b) one can see that the terminal spin currents I_1^y and I_2^y in the two leads have opposite signs, which means that the terminal spin currents with spin parallel to the y axis will flow out of the SO coupled region in both leads and hence are non-conservative (i.e., the spin current flowing into the SO coupled region does not equal to the spin current flowing out of the same region). Similarly one can show

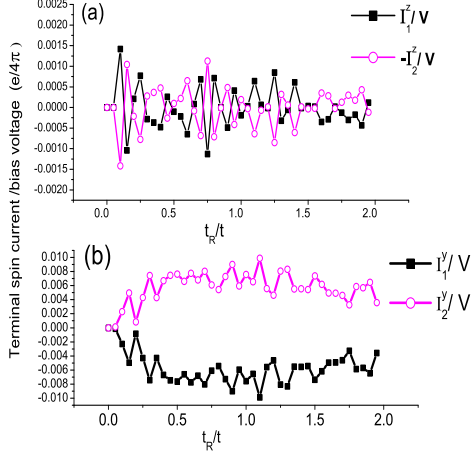


FIG. 6: (Color online)(a) The terminal spin currents I_1^z and $-I_2^z$ (divided by the voltage) as a function of the Rashba SO coupling strength t_R (in units of t). (b) The terminal spin currents I_1^y and I_2^y (divided by the voltage) as a function of the Rashba SO coupling strength t_R . The figures show that the terminal spin currents I_1^z and I_2^z in the two leads have the same signs and the terminal spin currents I_1^y and I_2^y in the two leads have opposite signs. (Note that for clarity a minus sign is added before I_2^z in Fig.6(a)). The parameters used are given in the text or shown in the figures.

that the terminal spin currents with spin parallel to the x axis have also opposite signs in the two leads, similar to the case shown in Fig.6(b). This simple example illustrates explicitly that the terminal spin currents in a multi-terminal mesoscopic SO coupled system are non-conservative in general. It should be stressed that this non-conservation of terminal spin currents is not caused by the use of an improper definition of spin current but is intrinsic to spin-dependent transports in mesoscopic SO coupled systems. In fact, in our calculations we did not involve the controversial issue of what is the correct definition of spin current in the SO coupled region at all, so the ambiguities that may be caused by the use of an improper definition of spin current to the SO coupled region have been eliminated in our calculations.

V. CONCLUSION

In summary, based on a scattering wave function approach, in this paper we have studied theoretically some symmetry properties of spin currents and spin polarizations in a multi-terminal mesoscopic structure in which a spin-orbit coupled system is contacted to several ideal and nonmagnetic external leads. Some interesting new results were obtained based on the symmetry analysis of spin currents and spin polarizations in such a multi-terminal mesoscopic structure. First, we showed that in

the equilibrium state no finite spin polarizations can exist both in the leads and in the central SO coupled region and also no finite equilibrium terminal spin currents can survive. Second, we showed that the lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of a ballistic two-dimensional electron gas with both Rashba and Dresselhaus SO coupling may be non-antisymmetric in general, which implies that some cautions may need to be taken when attributing the nonequilibrium lateral edge spin accumulation induced by a longitudinal charge current in a thin strip of such a system to a spin Hall effect, especially in the mesoscopic regime. Finally, by use of a typical two-probe structure as the example, we showed explicitly that the nonequilibrium terminal spin currents in a multi-terminal mesoscopic SO coupled system may be non-conservative in general. Some symmetry properties discussed in the present paper might also be helpful for clarifying some controversial issues related to the study of spin-dependent transports in macroscopic SO coupled systems.

Acknowledgments

Y. J. Jiang was supported by the Natural Science Foundation of Zhejiang province (Grant No.Y605167). L. B. Hu was supported by the National Science Foundation of China (Grant No.10474022) and the Natural Science Foundation of Guangdong province (No.05200534).

APPENDIX A: SOME DETAILS FOR THE DERIVATIONS OF THE SCATTERING AMPLITUDES AND THE TRANSMISSION PROBABILITIES

In this appendix we give some details on how to derive the scattering amplitudes and the transmission probabilities from the scattering wave function approach introduced in Sec.II. For convenience of notation, we arrange the scattering wave function $\psi^{pm\sigma}(\mathbf{r}_i)$ inside the SO coupled region into a column vector Ψ_s whose dimension is $2N$ (N is the total number of lattice sites in the SO coupled region) and arrange the scattering amplitudes $\phi_{qn\sigma'}^{pm\sigma}$ into a column vector Φ whose dimension is $2M$ ($M = \sum_p N_p$ and N_p is the width of lead p). Substituting Eqs.(3-4) into Eqs.(5a-5b) and making use of the orthogonality relations for the transverse modes in the leads, one can show that the two column vectors Ψ_s and Φ will satisfy the following relations:

$$\mathbf{A}\Psi_s = \mathbf{b} + \mathbf{B}\Phi, \mathbf{C}\Phi = \mathbf{d} + \mathbf{D}\Psi_s, \quad (\text{A1})$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are four rectangular matrices with the dimensions of $2N \times 2N$, $2N \times 2M$, $2M \times 2M$, and $2M \times 2N$, respectively; \mathbf{b} and \mathbf{d} are two column vectors with the dimensions of $2N$ and $2M$, respectively.

The elements of these matrices and column vectors can be written down explicitly as

$$\begin{aligned}
\mathbf{A} &= \mathbf{EI} - H_{sys}, \\
\mathbf{B}(n_{p''y}\sigma'', p'm'\sigma') &= -\delta_{p'',p'}\delta_{\sigma'',\sigma'}t_{p's}\chi_{m'}^{p'}(y_{p''})e^{ik_{m'}^{p'}}, \\
\mathbf{D}(p'm'\sigma', n_{p''y}\sigma'') &= -\delta_{p'',p'}\delta_{\sigma'',\sigma'}t_{p's}\chi_{m'}^{p'}(y_{p''}), \\
\mathbf{C}(p'm'\sigma', p''m''\sigma'') &= -\delta_{p'',p'}\delta_{\sigma'',\sigma'}\delta_{m''m'}t_{p'}, \\
\mathbf{b}(n_{p'y}\sigma') &= -\delta_{pp'}\delta_{\sigma\sigma'}t_{ps}\chi_m^p(y_{p'})e^{-ik_m^p}, \\
\mathbf{d}(p'm'\sigma') &= \delta_{pp'}\delta_{mm'}\delta_{\sigma\sigma'}t_p,
\end{aligned} \tag{A2}$$

where \mathbf{I} stands for the identity matrix. The indices for leads, transverse modes, lattice sites and spins can take all possible values. (For simplicity of notation, we have used simply a symbol $n_{p'y'}$ to denote a boundary lattice site in the SO coupled region which is connected directly to a boundary lattice site $\mathbf{x}'_{p'} = (1, y'_{p'})$ in lead p' .) Eq.(A1) is just a compact form of the match conditions (5a-5b) on the borders between the leads and the SO coupled region, from which both the scattering amplitudes $\{\phi_{qn\sigma'}^{pm\sigma}\}$ and the transmission probabilities $\{T_{q\sigma'}^{p\sigma}\}$ can be obtained readily.

To derive a compact formula for the transmission probabilities between two leads, we define an auxiliary matrix $\Sigma^R \equiv \mathbf{BC}^{-1}\mathbf{D}$. By use of Eq.(A2) the matrix elements of Σ^R can be written down readily as following,

$$\Sigma^R(n_{p'y_1}\sigma', n_{p'y_2}\sigma') = -\sum_{m'} \frac{t_{p's}^2}{t_{p'}} \chi_{m'}^{p'}(y_1) \chi_{m'}^{p'}(y_2) e^{ik_{m'}^{p'}}, \tag{A3}$$

and all other matrix elements not shown explicitly above are zero. With the help of this auxiliary matrix, from Eq.(A1) one can get that

$$\Psi_s = (\mathbf{A} - \Sigma^R)^{-1}(\mathbf{b} + \mathbf{BC}^{-1}\mathbf{d}) = G^R \mathbf{g}, \tag{A4}$$

where \mathbf{g} is a column vector defined by $\mathbf{g} \equiv \mathbf{b} + \mathbf{BC}^{-1}\mathbf{d}$ and G^R is a matrix defined by $G^R \equiv (\mathbf{A} - \Sigma^R)^{-1} =$

$[\mathbf{EI} - H_{sys} - \Sigma^R]^{-1}$, which is just the usual retarded Green's function. By use of Eq.(A2), the elements of the column vector \mathbf{g} can also be written down readily as following,

$$\mathbf{g}(n_{p'y}\sigma') = 2i\delta_{pp'}\delta_{\sigma\sigma'}t_{ps}\sin(k_m^p)\chi_m^p(y). \tag{A5}$$

By substituting Eq.(A4) into Eq.(A1) one gets that $\Phi = \mathbf{C}^{-1}\mathbf{d} + \mathbf{C}^{-1}\mathbf{D}G^R\mathbf{g}$. Inserting Eq.(A5) into this formula and making use of Eq.(A2), one can show readily that the scattering amplitudes $\phi_{qn\sigma'}^{pm\sigma}$ will be given by

$$\begin{aligned}
\phi_{p'm'\sigma'}^{pm\sigma} &= -\delta_{pp'}\delta_{mm'}\delta_{\sigma\sigma'} + 2it_{p'}^{-1}t_{p's} \sum_{y_p, y_{p'}} t_{ps} \sin(k_m^p) \\
&\times \chi_{m'}^{p'}(y_{p'}) G_{\sigma'\sigma}^R(n_{y_{p'}}, n_{y_p}) \chi_m(y_p).
\end{aligned} \tag{A6}$$

The total transmission probability of a conduction electron from lead p (with spin index σ) to lead p' (with spin index σ') is defined by $T_{p'\sigma'}^{p\sigma} = \sum_{m, m'} \left| \phi_{p'm'\sigma'}^{pm\sigma} \right|^2 \frac{v_{p'm'}}{v_{pm}}$, where $v_{p'm'} = \frac{1}{\hbar} 2t_{p'} \sin(k_{m'}^{p'})$ is the longitudinal velocity of the mode m' in lead p' . Substituting Eq.(A6) into this formula, for $p \neq p'$ one can get that

$$T_{p'\sigma'}^{p\sigma} = Tr(\Gamma^p G_{\sigma\sigma'}^A \Gamma^{p'} G_{\sigma'\sigma}^R), \tag{A7}$$

where $G_{\sigma\sigma'}^R$ and $G_{\sigma\sigma'}^A (\equiv G_{\sigma'\sigma}^{R\dagger})$ are the spin-resolved retarded and advanced Green's functions, respectively, and $\Gamma^p(y_p, \bar{y}_p)$ is defined by

$$\Gamma^p(y_p, \bar{y}_p) = \sum_m \left(\frac{t_{ps}}{t_p} \right)^2 \chi_m(y_p) v_{pm} \chi_m(\bar{y}_p). \tag{A8}$$

The transmission probabilities given by Eq.(A7) have exactly the same form as was obtained by the usual Green's function approach³³.

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